

Cosmological attractor inflation from the RG-improved Higgs sector of finite gauge theory

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Abstract. The possibility to construct an inflationary scenario for renormalization-group improved potentials corresponding to the Higgs sector of finite gauge models is investigated. Taking into account quantum corrections to the renormalization-group potential which sums all leading logs of perturbation theory is essential for a successful realization of the inflationary scenario, with very reasonable parameter values. The inflationary models thus obtained are seen to be in good agreement with the most recent and accurate observational data. More specifically, the values of the relevant inflationary parameters, n_s and r , are close to the corresponding ones in the R^2 and Higgs-driven inflation scenarios. It is shown that the model here constructed and Higgs-driven inflation belong to the same class of cosmological attractors.

Keywords: cosmological attractors, inflation, renormalization group, non-minimal coupling

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1 Introduction

The existence of an extremely short stage of accelerated expansion in the very early Universe (inflation) provides a simple explanation of astronomical data, including the fact that, at cosmological distances, the Universe is approximately isotropic, homogeneous, and spatially flat. Models of cosmic inflation yield accurate quantitative predictions for a number of observable quantities [1–3]. It is known that scalar fields play an essential role in the current description of the evolution of the Universe at a very early epoch [4–7]. Modified gravity inflationary models [8–10], for example, R^2 inflation [11, 12], which can be considered as generic General Relativity models with additional scalar fields, are quite popular as well.

The confirmed discovery of the Higgs boson at the Large Hadron Collider (CERN) has initiated an intense research activity with the aim to understand the cosmological implications of this truly fundamental scalar field. A really crucial issue in this respect is the possibility to describe inflation using particle physics [13–15], as the Standard Model of elementary particles itself [16–20], or either supersymmetric models [21–23], or even non-supersymmetric grand unified theories (GUTs) [5, 13, 24].

Quantum field theory in curved space-time is an unavoidable, extremely important concept at the very early universe, where curvature is large and typical energies are very high. In this situation, quantum effects necessarily modify the gravitational action and may cause inflation to occur, as well as other interesting astrophysical phenomena. For instance, it is well-known that quantum field theories in curved space-time lead to curvature-induced phase transitions (for a detailed description, see [25–27]). More precisely, these curvature-induced phase transitions may simply and naturally explain the origin of inflation itself, especially, by taking into account the point of view that the inflaton might be nothing else but the Higgs field.

As discussed in [25, 26], the most completed description of curvature induced phase transitions can be given in terms of the summation of all leading-logs of quantum field theory, when one considers this issue within the renormalization-group improved effective potential perspective (see [27, 28]). Indeed, in this case, the corresponding RG-improved effective potential goes far beyond the one-loop approximation. Let us stress, once more, that these phase transitions are in fact very important in the early universe. In particular, a large number of models of the inflationary universe [3] are based on first-order phase

transitions, which took place during the reheating phase of the Universe, in the epoch when the grand unification description is applied [5]. Hence, in the absence of a consistent theory of quantum gravity, all classical theories should better be treated as quantum field theories in curved space-time, as extensively discussed in [25, 27]. In fact, some recent results by the Planck collaboration [29, 30] seem to point towards the GUT scale, what is quite a remarkable connection between GUTs and inflation. And, in this context, GUTs ought to be treated as quantum field theories in the curved space-time corresponding to the very early universe. The calculation of beta-functions which was done in previous works (as in the book by [25]) is based on the use of dimensional regularization and, hence, it does not depend on any explicit cut-off choice. We consider that Quantum Gravity effects are less relevant in this approach, since we work below the Planck scale.

As a consequence, it is natural to start with the issue of the renormalization-group improved effective potential for an arbitrary renormalizable massless gauge theory in curved space-time [27]. Note that it is enough to work in the linear curvature approximation, because these linear curvature terms are expected to give dominant contribution in the discussion of the inflationary effective potential corresponding to GUTs. Observe also that we work with Higgs-like inflation where the scalar potential receives quantum corrections. In this case, the linear curvature term is the leading one, as compared with the R^2 term, which is also induced by quantum corrections. Indeed, it is known that Starobinsky's R^2 inflation is classically equivalent to Higgs inflation, because both models generate a specific exponential potential in the Einstein frame. Hence, the model under discussion here, constrained by a scalar potential and a linear curvature term, is also classically equivalent to a specific $F(R)$ gravity theory. By generalizing the flat space-time Coleman–Weinberg potential [31], the explicit form of the renormalization group (RG) improved effective potentials in curved space finite gauge models were obtained in [28]. The occurrence of curvature-induced phase transitions was also studied there in detail.

It is well-known that in high energy physics there exist so-called finite (gauge) theories. There are two classes of such theories. The first class are the theories in which, by some reason, the corresponding coupling constants are not renormalized up to some loop. For instance, those could be one-loop finite theories. This simply means that the corresponding one-loop beta-functions are zero but, at the next order in the loop expansion the beta-functions do receive corrections. The second, and more important, class of finite theories is usually a consequence of supersymmetry. Indeed, a few supersymmetric theories have been proven to be finite to all orders of perturbation theory ($N = 4$ super Yang–Mills theory [32] being a well known example). However, it is a fact that supersymmetric theories are not the only ones which can be finite at the one- or two-loop levels; different GUTs have been proposed which turn out to yield finite models, too. Asymptotically finite GUTs [33] are generalizations of the concept of a finite theory, in which the zero charge problem is absent. Indeed, both in the UV and in the IR limits the effective coupling constants tend to some constant values (corresponding to finite phases). However, all the above remarks are about finite theories in flat space. When we consider such finite theories in curved spacetime the situation changes qualitatively. The point is that even the finite gauge theories to all loops are not finite in curved spacetime. There are two sectors where the theory ceases to be finite. First of all, in order to make the theory multiplicatively-renormalizable in curved spacetime one has to add to the matter Lagrangian the so-called Lagrangian of the external gravitational field (vacuum polarization terms) (see the book [25]). There appear corresponding couplings and beta-functions for this external field Lagrangian. The leading contribution to these beta-

functions is proportional to a number of fields (it maybe read off from the coefficients of the conformal anomaly) and is defined by the structure of the free matter Lagrangian. There is no way to make these external couplings beta-functions to be zero. The second sector where non-zero one-loop beta functions appear is the non-minimal coupling of scalar field with curvature. Again, the corresponding beta-function cannot be made zero for arbitrary values of the non-minimal coupling ξ . Hence, even being a theory finite in flat space-time, it becomes non-finite (or finite only over part of the coupling constants) in curved spacetime. Due to multiplicative renormalizability of such theory, one can apply the standard methods of renormalization group for such partly finite gauge theory in order to get the effective potential. In this way, we expect to find a high-energy physics motivation for the class of exponential potentials to describe the inflationary universe. (Note that one of the main problems of inflationary cosmology is the fact that many inflationary potentials are taken ad hoc, without any physical justification for the corresponding choice). Furthermore, it maybe expected that some flat-space finite SUSY theories (like $N = 4$ super Yang–Mills theory) are relevant precisely at the very early universe when inflation starts.

Very recent observations [29, 30, 34] (see also [35]) result in important restrictions on existing inflationary models. The temperature data of the Planck full mission and the first release of polarization data on large angular scales [30], constraint the spectral index of curvature perturbations to be

$$n_s = 0.968 \pm 0.006 \quad (68\% \text{ c.l.}), \quad (1.1)$$

and the upper bound on the tensor-to-scalar ratio, as

$$r < 0.11 \quad (95\% \text{ c.l.}). \quad (1.2)$$

The high-precision measurements performed by the Planck survey show that non-Gaussian perturbations are quite small, what makes single-field inflationary models become more realistic. At the same time, the predictions of the simplest inflationary models with a minimally coupled scalar field lead to rather large values of the tensor-to-scalar ratio of the density perturbations r , and are therefore ruled out by recent Planck data [29, 30]. Such inflationary scenarios can be improved by adding a tiny non-minimal coupling of the inflaton field to gravity [36, 37]. This is not so artificial as it might seem at first sight, since one should note that generic quantum corrections to the action of the scalar field minimally coupled to gravity do include a non-minimally coupling term [38], proportional to $R\phi^2$ and, as is clear, quantum corrections must definitely be taken into account at the inflationary scales. The non-minimal $R\phi^2$ term is always induced by quantum corrections and its presence assures the multiplicative renormalizability of scalar theory in curved space-time. Inflationary models with the Ricci scalar multiplied by a function of the scalar field are being intensively studied in cosmology [6, 13, 14, 16–20, 36, 37, 39–50].

There are several inflationary models which are described by the renormalization group improved effective action [18, 19, 45, 46, 51, 52]. In our paper [45], we showed that both for scalar electrodynamics and for the $SU(5)$ RG-improved models,¹ inflationary scenarios are possible and they are in good agreement with the most recent astronomical data [29, 35], provided some reasonable values are taken for the parameters. In [45] we checked the possibility to construct inflationary models using RG-improved effective potentials, considering inflation based on de Sitter solutions, their instability providing a graceful exit from inflation.

¹The corresponding potentials had been proposed in [27, 28].

In the present paper we make a step forward and show that RG-corrections can *generate* a Hilbert–Einstein term in the action. In other words, we do not need to include this term by hand, as is done in Higgs-driven inflation [16–20], since we get it naturally, as part of the quantum corrections to the induced gravity term.

There are many different models of inflation, some of them giving very close predictions to each other, for the observable inflationary parameters. For instance, the two different models, R^2 Starobinsky inflation [11] and Higgs-driven inflation [16, 17], yield very similar values for the spectral index of the curvature perturbations n_s and for the tensor-to-scalar ratio of the density perturbations r , respectively. In both models [53], one gets approximately

$$n_s \simeq 1 - \frac{2}{N_e}, \quad r \simeq \frac{12}{N_e^2}, \quad (1.3)$$

where N_e is the number of e-foldings during inflation. This number must be matched with the appropriate normalization of the data set and with the cosmic history. For the standard interval $50 \leq N_e \leq 65$ formula (1.3) gives suitable values of n_s and r . In recent papers [47–50, 54–58] it has been shown that there are several classes of inflationary models such that all models within a given class predict the same values of n_s and r in the leading approximation in $1/N_e$.

In the present paper we consider models with non-minimally coupled scalar field and finite gauge RG-improved potentials. We show that for the finite gauge models, the inflationary scenario can be realized without having an exact de Sitter solution, but indeed from a quasi-de Sitter solution with a slowly changing Hubble parameter. We calculate the corresponding spectral index of curvature perturbations, the tensor-to-scalar ratio of density perturbations, and the running of the spectral index in the new model and show that an inflationary model with this potential is truly compatible with the most recent cosmological data. We will see that our model here yields the same inflationary parameters as the R^2 and Higgs-driven inflationary models and does belong to the same class of cosmological attractors as Higgs-driven inflation and R^2 Starobinsky inflation. We also show that the cosmological attractor method is useful to get inflationary parameters for the model considered.

The paper is organized as follows. In Sect. 2 we recall the basic formulae to explore inflationary models with non-minimally coupled scalar fields. Sect. 3 is devoted to the construction of inflationary models based on the RG-improved Higgs sector of the finite gauge model. In Sect. 4 a comparison is carried out of the inflationary scenarios here obtained with the general class of inflationary models known as α -attractors. In Sect. 5 we compare the model considered with the R^2 inflationary scenario. Finally, the last section is devoted to conclusions and prospects for future research.

2 Inflationary models with non-minimal coupling

Let us consider a gravity model with a non-minimally coupled scalar field, described by the action

$$S = \int d^4x \sqrt{-g} \left[U(\phi) R - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right], \quad (2.1)$$

where $U(\phi)$ and $V(\phi)$ are differentiable functions of the scalar field ϕ , g is the determinant of the metric tensor $g_{\mu\nu}$, and R is the scalar curvature.

In a spatially flat FLRW universe, with the interval

$$ds^2 = -dt^2 + a^2(t) (dx_1^2 + dx_2^2 + dx_3^2),$$

the Friedmann equations, derived by variation of the action (2.1), have the following form [59]:

$$6UH^2 + 6\dot{U}H = \frac{1}{2}\dot{\phi}^2 + V, \quad (2.2)$$

$$2U(2\dot{H} + 3H^2) + 4\dot{U}H + 2\ddot{U} = -\frac{1}{2}\dot{\phi}^2 + V, \quad (2.3)$$

where the Hubble parameter is the logarithmic derivative of the scale factor: $H = \dot{a}/a$ and differentiation with respect to time t is denoted by a dot. Variation of the action (2.1) with respect to ϕ yields

$$\ddot{\phi} + 3H\dot{\phi} + V' = 6(\dot{H} + 2H^2)U', \quad (2.4)$$

where the prime denotes derivative with respect to the scalar field ϕ .

The standard way to calculate the parameters of inflation is to perform a conformal transformation and consider the model in the Einstein frame (see, for instance [18]). There is an ongoing discussion about the physical equivalence of these two frames [60], in special on the equivalence of the corresponding quantum theories [20, 61, 62] (at the classical level the issue seems to be clear by now). In our paper we consider the Jordan frame to be the physical one. For this reason, we calculate quantum corrections in the Jordan frame. The calculation of the β functions in the Einstein frame potential may, therefore, yield a different result. On the other hand, once the quantum corrections have been obtained, one can use the Einstein frame to get the inflationary parameters. This is possible because of the quasi de Sitter expansion occurring during inflation. Indeed, it has been shown [44], that in the case of quasi de Sitter expansion there is no difference between the inflationary parameters calculated either in the Jordan frame directly, or in the Einstein frame, after the corresponding conformal transformation. In Sect. 4 we will show that the condition (4.1) plays an important role for the equivalence of the Jordan and Einstein frames during inflation.

Let us now perform the conformal transformation

$$\tilde{g}_{\mu\nu} = 16\pi M_{\text{Pl}}^{-2}U(\phi)g_{\mu\nu},$$

where the metric in the Einstein frame is marked with a tilde, and M_{Pl} is the Planck mass.

After this transformation, we get a model for a minimally coupled scalar field, described by the following action

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2}{16\pi} R(\tilde{g}) - \frac{M_{\text{Pl}}^2}{32\pi U} \left[1 + \frac{3U'^2}{U} \right] \tilde{g}^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V_E(\phi) \right], \quad (2.5)$$

where the potential in the Einstein frame is

$$V_E = \frac{M_{\text{Pl}}^4 V(\phi)}{256\pi^2 U^2(\phi)}. \quad (2.6)$$

Many inflationary models are based upon the possibility of a slow evolution of some scalar field ϕ . To calculate parameters of inflation that can be tested via observations, we use the slow-roll approximation [6, 63] (see also [44, 64]). During inflation, the slow-roll parameters ϵ , η and ζ should remain to be less than one. Note that we do not introduce

a new scalar field when considering the Einstein frame action, because it is suitable [45] to express slow-roll and inflationary parameters in terms of the initial scalar field ϕ :

$$\epsilon = \frac{M_{\text{Pl}}^2 (V'_E)^2}{16\pi V_E^2 Q}, \quad \eta = \frac{M_{\text{Pl}}^2}{8\pi V_E Q} \left[V_E'' - \frac{V'_E Q'}{2Q} \right], \quad (2.7)$$

$$\zeta^2 = \frac{M_{\text{Pl}}^4 V'_E}{64\pi^2 V_E^2 Q^2} \left[V_E''' - \frac{3V_E'' Q'}{2Q} - \frac{V'_E Q''}{2Q} + \frac{V'_E (Q')^2}{Q^2} \right], \quad (2.8)$$

where

$$Q = \frac{M_{\text{Pl}}^2 (U + 3U'^2)}{16\pi U^2}.$$

The number of e-foldings in slow-roll inflation is given by the integral [18]

$$N_e(\phi) = \frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_{\text{end}}}^{\phi} \left| \frac{V_E}{V'_E} \right| Q d\tilde{\phi} = \frac{2\sqrt{\pi}}{M_{\text{Pl}}} \int_{\phi_{\text{end}}}^{\phi} \frac{\sqrt{Q}}{\sqrt{\epsilon}} d\tilde{\phi}, \quad (2.9)$$

where ϕ_{end} is the value of the field at the end of inflation, defined by the condition $\epsilon = 1$ at $\phi = \phi_{\text{end}}$. The ratio r of squared amplitudes of tensor and scalar perturbations, the scalar spectral index of the primordial curvature fluctuations n_s , and the associated running of the spectral index α_s , are given, to very good approximation, by

$$r = 16\epsilon, \quad n_s = 1 - 6\epsilon + 2\eta, \quad \alpha_s \equiv \frac{dn_s}{d \ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\zeta^2. \quad (2.10)$$

3 Finite gauge models

Let us consider a massless finite or massless asymptotically finite GUT. In flat space-time quantum corrections to the classical potential are either absent or highly suppressed asymptotically. However, in curved space-time in such type of models the coupling parameter ξ corresponding to the non-minimal scalar-gravitational interaction receives quantum corrections [65] (for a general review, see [25]). In the RG-improvement scheme the corresponding RG-parameter depends on the scalar field. The general structure of the one-loop effective coupling parameter $\xi(\phi)$ for “finite” theories in curved space-time has been obtained in [65]. In the one-loop approximation, the RG-equation for the parameter $\xi(\phi)$ is

$$\frac{d}{d\vartheta} \xi(\vartheta) = \left(\xi(\vartheta) - \frac{1}{6} \right) C g^2, \quad (3.1)$$

where $\vartheta(\phi) = \frac{1}{2} \ln(\phi^2/\mu^2)$, C is a nonzero constant, μ is a parameter that defines the GUT scale. A parameter $g^2 \ll 1$ (it should be clear that g^2 is not the square of the determinant of the metric $g_{\mu\nu}$). Equation (3.1) has the following solution:

$$\xi(\vartheta) = \frac{1}{6} + \left(\xi_0 - \frac{1}{6} \right) e^{C g^2 \vartheta}, \quad (3.2)$$

with an integration constant ξ_0 . If $C > 0$, then $|\xi(\vartheta)| \rightarrow \infty$ (non-asymptotical conformal invariance) in the UV limit ($\vartheta \rightarrow \infty$). In the models with $C < 0$, one gets $\xi(\vartheta) \rightarrow 1/6$ (asymptotically conformal invariance).

In the model considered, the tree-level functions of the scalar field V and U are [27]:

$$V^{(0)} = \tilde{a}\lambda\phi^4, \quad U^{(0)} = b\xi\phi^2, \quad (3.3)$$

where \tilde{a} and b are positive constants and ξ is the conformal coupling. The corresponding potential in the Einstein frame V_E is a constant and is not suitable for inflation. To realize the inflationary scenario with a graceful exit from inflation we use the RG-improved potentials². The functions U and V , obtained in the linear curvature approximation (see [28, 45] for details), are given by

$$V = \tilde{a}\kappa_1 g^2 f^4(\vartheta)\phi^4, \quad U = b\xi(\vartheta)f^2(\vartheta)\phi^2, \quad (3.4)$$

where $f(\vartheta) = \exp(-C_1 g^2 \vartheta)$, κ_1 , and C_1 are some constants which depend on the gauge parameter and on the features of the theory, and $\xi(\vartheta)$ is given by (3.2). Thus,

$$f(\vartheta) = \psi^{-C_1 g^2}, \quad \xi(\vartheta) = \frac{1}{6} + \left(\xi_0 - \frac{1}{6}\right) \psi^{C g^2},$$

where $\psi = \phi/\mu$ is a dimensionless field. Observe that, in finite GUTs, the connection between the parameters C and C_1 is not specified.

From (3.4) we get the potential in the Einstein frame

$$V_E(\phi) = \frac{9M_{\text{Pl}}^4 \tilde{a}\kappa_1 g^2}{64\pi^2 b^2 (1 + (6\xi_0 - 1) \psi^{C g^2})^2}. \quad (3.5)$$

Note that V_E does not depend on C_1 , whereas Q depends on it:

$$Q = \frac{3M_{\text{Pl}}^2 \left(b \left[(2 + (C - 2C_1)g^2)(6\xi_0 - 1)\psi^{C g^2} + 4 - 4C_1 g^2 \right]^2 \psi^{-2C_1 g^2} + 6 + 6(6\xi_0 - 1)\psi^{C g^2} \right)}{16\pi b \psi^{-2C_1 g^2} \phi^2 (1 + (6\xi_0 - 1)\psi^{C g^2})^2}.$$

In [45] we studied de Sitter solutions in cosmological models with renormalization-group improved effective potentials for some finite gauge theories. We obtained that there is no de Sitter solutions for $C \neq 0$. In the case $C = 0$ the potential V_E is a constant. So, this case is not suitable for inflation.

In this paper we consider the possibility to realize an inflationary scenario without an exact de Sitter solution. We see that V_E' tends to zero for $\phi \rightarrow \infty$. When $C < 0$ the potential V_E tends to a maximal value for $\phi \rightarrow \infty$ (see Fig. 1) and the model has a quasi-de Sitter solution, in other words, an approximately constant Hubble parameter for large ϕ . So, we can expect a slow-roll behavior of the scalar field.

The functions U and V , given by (3.4), look complicated but for some values of the constants C and C_1 they are actually simple and exhibit very interesting properties. Let us first consider the function $U(\phi)$. The function $U^{(0)}$ corresponds to the induced gravity, in other words, the term proportional to $\phi^2 R$ plays the role of the Hilbert–Einstein term in the action. It is quite interesting that for some values of the parameters the renormalization-group corrections can yield the standard Hilbert–Einstein term. Indeed, if

$$C_1 = \frac{1}{g^2} + \frac{C}{2}, \quad (3.6)$$

²The main renormalization-group formulae for models in curved space-time are given in [27, 28].

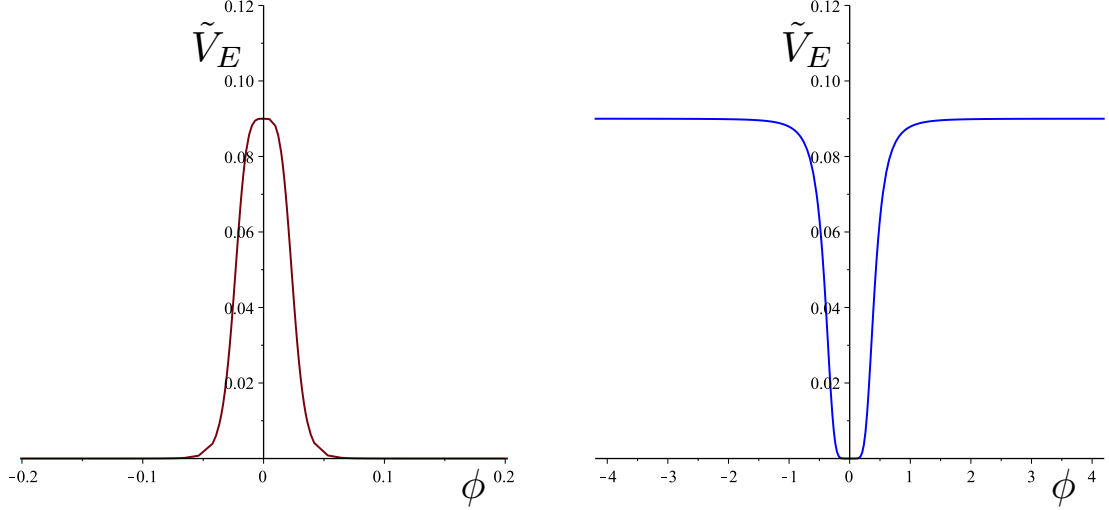


Figure 1. The function $\tilde{V}_E \equiv \frac{64\pi^2}{M_{\text{Pl}}^4} V_E$ at $C = 4/g^2$ (left) and $C = -4/g^2$ (right). Other parameters have the following values: $\xi_0 = 20$, $\tilde{a} = 1$, $b = 1$, $\kappa_1 = 1$, $g = 0.1$, $\mu = 0.1$.

then function U acquires the following form

$$U = \frac{1}{6} b \mu^2 \left(6\xi_0 - 1 + \psi^{-Cg^2} \right). \quad (3.7)$$

Thus, the Hilbert–Einstein term arises as a renormalization-group correction. If the condition (3.6) is satisfied, then we also obtain

$$V = \tilde{a} \kappa_1 g^2 \mu^4 \psi^{-2Cg^2} = \frac{36}{b^2} \tilde{a} g^2 \kappa_1 \left(U - \frac{b \mu^2}{6} (6\xi_0 - 1) \right)^2. \quad (3.8)$$

This relation between the functions V and U means that the corresponding inflationary model is a cosmological attractor (see the next section). Dynamics of similar cosmological models, that include the Hilbert–Einstein curvature term and the monomial of the scalar field non-minimally coupled to gravity, have been considered in [69].

Let us analyze possible inflationary scenarios in this case. As we see from Fig. 1, the case with a negative C is more suitable for inflation. Using (3.7) and (3.8), we get the inflationary parameters:

$$\begin{aligned} \epsilon &= \frac{4\mu^4 b^2 (6\xi_0 - 1)^2 U'^2}{(b\mu^2(6\xi_0 - 1) - 6U)^2 (U + 3U'^2)} = \frac{8bn^2(6\xi_0 - 1)^2}{6n^2 b \psi^{4n} + 3\psi^{2n+2} + 3(6\xi_0 - 1)\psi^2}. \\ \eta &= \frac{4nb(6\xi_0 - 1)}{3(2bn^2\psi^{4n} + \psi^{2n+2} + (6\xi_0 - 1)\psi^2)^2} \left[-4bn^3\psi^{6n} - (n+1)\psi^{4n+2} \right. \\ &\quad \left. + 4n^3b(6\xi_0 - 1)\psi^{4n} + (3n-2)(6\xi_0 - 1)\psi^{2+2n} + (6\xi_0 - 1)^2(4n-1)\psi^2 \right], \end{aligned} \quad (3.9)$$

where $n = -Cg^2/2$.

The finite gauge model here considered depends on eight parameters³, but slow-roll parameters ϵ and η depend only on four parameters, namely n , b , ξ_0 , and μ . The relation between them can be obtained from (3.7), because the function U should reproduce $M_{\text{Pl}}^2/(16\pi)$

³Two of them are connected by (3.6).

at low energy. Thus,

$$b = \frac{3}{8\pi(6\xi_0 - 1)\mu^2} M_{\text{Pl}}^2. \quad (3.10)$$

If $\psi^2 \gg 1$ during inflation, then slow-roll parameters do not depend on b in first approximation. Therefore, we can fix b , using (3.10), and construct suitable inflationary scenarios by choosing the parameters n , μ , and ξ . We consider a GUT model and thus a reasonable value of μ is about $10^{-3} M_{\text{Pl}}$. At the same time, it is very interesting that we can actually get a suitable inflationary scenario with approximately the same values for the inflation parameters, for different values of the model parameters ξ_0 and μ . Note that the parameter μ appears in dimensionless combination ϕ/μ only.

Let us assume that $1/\psi^2$ is a small parameter and n is a natural number. The case $n = 1$ is special, because in this case $C_1 = 0$, the case $n = 2$ will be consider below in detail. Let us consider the case $n > 2$. Such as

$$\psi^2 \gg 1, \quad (3.11)$$

we obtain from (3.9):

$$\epsilon \approx \frac{4}{3} (6\xi_0 - 1)^2 \psi^{-4n}, \quad \eta \approx -\frac{4}{3} (6\xi_0 - 1) \psi^{-2n}. \quad (3.12)$$

In this approximation $\epsilon|_{\psi=\psi_{\text{end}}} \approx 1$ at

$$\psi_{\text{end}} \equiv \frac{\phi_{\text{end}}}{\mu} \approx \left[\frac{2}{\sqrt{3}} (6\xi_0 - 1) \right]^{\frac{1}{2n}}. \quad (3.13)$$

The inflation parameters and the number of e-foldings can be approximated as follows

$$n_s \approx 1 - \frac{8}{3} (6\xi_0 - 1) \psi_N^{-2n}, \quad r \approx \frac{64}{3} (6\xi_0 - 1)^2 \psi_N^{-4n}, \quad \alpha_s \approx -\frac{32}{9} (6\xi_0 - 1)^2 \psi_N^{-4n},$$

$$N_e \approx \frac{3(\psi_N^{2n} - \psi_{\text{end}}^{2n})}{4(6\xi_0 - 1)}, \quad (3.14)$$

where ψ_N is the value of ψ at which the inflationary parameters are calculated. Using (3.13), we get

$$n_s \approx 1 - \frac{4}{\sqrt{3}} \Upsilon^{-2n}, \quad r \approx 16 \Upsilon^{-4n}, \quad \alpha_s \approx -\frac{8}{3} \Upsilon^{-4n}, \quad N_e \approx \frac{1}{2\sqrt{3}} (3\Upsilon^{2n} - 1),$$

where $\Upsilon = \psi_N/\psi_{\text{end}}$.

For $N_e \simeq 60$, we obtain that $\Upsilon^{2n} \gg 1$, hence, an approximated expression for N_e is

$$N_e \approx \frac{\sqrt{3}}{2} \Upsilon^{2n}.$$

We conclude that in our model, if the condition (3.11) is satisfied, then there exist the following relations between the inflation parameters and the e-folding number N_e :

$$n_s \approx 1 - \frac{2}{N_e}, \quad r \approx \frac{12}{N_e^2}, \quad \alpha_s \approx -\frac{2}{N_e^2}. \quad (3.15)$$

For example, at $N_e = 60$, formula (3.15) gives

$$n_s \approx \frac{29}{30} \approx 0.9667, \quad r \approx \frac{1}{300} \approx 0.0033333, \quad \alpha_s \approx -\frac{1}{1800} \approx -0.00055556. \quad (3.16)$$

We see that the expressions for r and n_s in (3.15) coincide with (1.3). Thus, we see that in order to get a inflationary model with inflationary parameters that are in good agreement with the observational data, it is sufficient to satisfy the conditions $\psi_N^{2n} \gg \psi_{end}^{2n}$ and (3.11) during inflation. This result will be explained in the next section with the help of the method the cosmological attractors [49].

Now, we show numerically that the suitable inflationary parameters can be obtained for a wide region of the finite gauge model parameters. Let us consider the case $n = 2$, that corresponds to

$$C_1 = -\frac{1}{g^2}, \quad C = -\frac{4}{g^2}, \quad U = \frac{b}{6}\mu^2(6\xi_0 - 1) + \frac{b}{6\mu^2}\phi^4, \quad V = \frac{\tilde{a}\kappa_1 g^2}{\mu^4}\phi^8. \quad (3.17)$$

Substituting (3.17), we obtain

$$V_E = \frac{9M_{\text{Pl}}^4 \tilde{a}\kappa_1 g^2 \phi^8}{64\pi^2 b^2 (\phi^4 + (6\xi_0 - 1)\mu^4)^2}, \quad Q = \frac{3M_{\text{Pl}}^2 [(6\xi_0 - 1)\mu^6 + \mu^2\phi^4 + 8b\phi^6]}{16\pi b (\phi^4 + (6\xi_0 - 1)\mu^4)^2},$$

and also

$$\epsilon = \frac{32b(6\xi_0 - 1)^2}{3\psi^2 [3(6\xi_0 - 1) + \psi^4 + 8b\psi^6]}. \quad (3.18)$$

Thus, we can analytically calculate the value of ψ that corresponds to the end of inflation: $\epsilon = 1$.

One can see that, for the values of the parameters given in Table 1, the corresponding values of the inflationary parameters n_s , r and α_s are in good agreement with the observational data (Eqs. (1.1) and (1.2)). Note that the parameters of inflation displayed in Table 1 and Table 2 were obtained through numerical calculation, without any approximation whatsoever. Parameter b is determined by (3.10) and the inflationary parameters depend on the number of e-foldings.

From Table 1 we see that the number of e-foldings N_e essentially influences the values of the inflation parameters, whereas their dependence on μ and ξ_0 is very small. The values of the inflationary parameters here obtained are close to the values one gets from the approximate formula (3.16). This means that a wide domain of parameters μ and λ are actually suitable for inflation. Even then, there are the following restrictions on these parameter. First, we consider $\xi > 1/6$, which corresponds to $b > 0$ and $U(\phi) > 0$ at all ϕ . To use the approximate formulae for the inflationary parameters we impose the condition (3.11), which should be satisfied at the point $\psi = \psi_N$. It is not easy to get the corresponding restriction on the model parameters. At the same time, the stronger condition $\psi_{end} \gg 1$ gives an explicit restriction on the parameter ξ_0 , due to (3.13). Note that this condition is actually sufficient. For example, for $\mu = 10^{-3}$ the inflation parameters are approximately the same for different values of ξ_0 (see Table 2). We observe that suitable values for the inflation parameters can be obtained even if $\psi_{end} < 1$.

To get a suitable inflationary scenario it is necessary to obtain inflationary parameters which are compatible with observation data, but this is not sufficient. Indeed, we must examine whether graceful exit from inflation can occur in the model here considered. It is

Table 1. Parameter values for the inflationary scenario at $n = 2$.

μ	ξ_0	ψ_{end}	N_e	ψ_N	n_s	r	$-\alpha_s$
$0.1M_{\text{Pl}}$	20	3.415	50	9.539	0.9613	0.0043	0.00076
			55	9.760	0.9650	0.0036	0.00063
			60	9.966	0.9675	0.0030	0.00053
			65	10.161	0.9700	0.0026	0.00045
$0.1M_{\text{Pl}}$	25	3.523	50	10.083	0.9612	0.0044	0.00076
			55	10.317	0.9650	0.0036	0.00063
			60	10.536	0.9675	0.0031	0.00052
			65	10.741	0.9700	0.0026	0.00045
$0.01M_{\text{Pl}}$	$5 \cdot 10^4$	23.961	50	67.813	0.9614	0.0043	0.00075
			55	69.364	0.9650	0.0035	0.00062
			60	70.926	0.9678	0.0030	0.00052
			65	72.199	0.9701	0.0026	0.00045
$0.001M_{\text{Pl}}$	1	1.550	50	4.346	0.9616	0.0042	0.00075
			55	4.445	0.9650	0.0035	0.00062
			60	4.538	0.9678	0.0030	0.00052
			65	4.626	0.9703	0.0025	0.00045

Table 2. Parameter values for the inflationary scenario at $n = 2$, $\mu = 0.001M_{\text{Pl}}$ and $N_e = 60$.

μ	ξ_0	ψ_{end}	N_e	ψ_N	n_s	r	$-\alpha_s$
$0.001M_{\text{Pl}}$	0.2	0.693	60	2.030	0.9678	0.0030	0.00052
$0.001M_{\text{Pl}}$	0.3	0.980	60	2.870	0.9678	0.0030	0.00052
$0.001M_{\text{Pl}}$	1	1.550	60	4.538	0.9678	0.0030	0.00052
$0.001M_{\text{Pl}}$	20	3.424	60	10.024	0.9678	0.0030	0.00052
$0.001M_{\text{Pl}}$	10^3	9.123	60	26.710	0.9678	0.0030	0.00052
$0.001M_{\text{Pl}}$	10^5	28.846	60	84.464	0.9679	0.0030	0.00052

easy to see that, during inflation, the inflaton moves from large to values of ϕ to $\phi = 0$, which corresponds to a minimum of the potential in the Einstein frame. Therefore, what we get is slow-roll inflation and subsequent oscillations of the inflaton near a minimum of the potential V_E . We also have shown that, during inflation, our model is close to a well-known inflationary model which has no problems with the exit from inflation. Taking into account that the inflationary scenario was naturally generated by quantum corrections, this is already a quite remarkable result.

4 Attractor behavior of the considered inflationary model

There are plenty of inflationary models and it is of interest to find the place of the model here discussed, and to compare it with other known models, what we are going to do next. In [47, 49] it has been shown that, if in the Jordan frame $V = c^2(U - U_0)^2$, where c and U_0 are some nonzero constants and V is proportional to ϕ with some positive power, then one finds a generalization of the Starobinsky potential in the Einstein frame, what has been called the $\alpha - \beta$ attractor model [49].

The idea of cosmological attractors is a very interesting one since it tries to select, among the huge number of inflationary models, a distinguished family of them which is not extremely dependent on the initial conditions and which, in a natural, quite generic setup, may give rise, with high probability, to inflation. It is based on the specific observation that, for many cosmological models with non-minimally coupled scalar fields, the following relation is satisfied:

$$1 \ll \frac{3U'^2}{U}. \quad (4.1)$$

In this approximation $Q \approx 3M_{\text{Pl}}^2 U'^2 / (16\pi U^2)$. Also, the action (2.5) that corresponds to the Einstein frame can be simplified to

$$\begin{aligned} S_E &\simeq \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2}{16\pi} R(\tilde{g}) - \frac{3M_{\text{Pl}}^2 U'^2}{32\pi U^2} \tilde{g}^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V_E(\phi) \right] \\ &\simeq \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2}{16\pi} R(\tilde{g}) - \frac{3M_{\text{Pl}}^2}{32\pi U^2} \tilde{g}^{\mu\nu} U_{,\mu} U_{,\nu} - \tilde{V}_E(U) \right], \end{aligned}$$

where $\tilde{V}_E(U) = V_E(\phi(U))$. In other words, one can consider U as a scalar field in the Einstein frame.

Let us assume that the functions U and V are connected by the following relation

$$V = c^2 (U - U_0)^2, \quad \Rightarrow \quad \tilde{V}_E = \frac{M_{\text{Pl}}^4 c^2}{256\pi^2} \left(1 - \frac{U_0}{U} \right)^2. \quad (4.2)$$

Also, we assume that the function U tends to the constant U_0 when $\phi \rightarrow 0$. One can check that indeed the functions U and V , given by (3.7) and (3.8), satisfy these conditions. From (3.8) and (3.10) we obtain that for the model considered

$$U_0 = \frac{M_{\text{Pl}}^2}{16\pi}, \quad c^2 = 256\pi^2 \tilde{a} g^2 \kappa_1 (6\xi_0 - 1)^2 \frac{\mu^4}{M_{\text{Pl}}^4}. \quad (4.3)$$

To get a scalar field with a standard kinetic term we express U as a function of a new scalar field, φ :

$$U = U_0 e^{\frac{4\sqrt{\pi}}{\sqrt{3}M_{\text{Pl}}}\varphi}. \quad (4.4)$$

Now, the action S_E reads

$$S_E \simeq \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\text{Pl}}^2}{16\pi} R(\tilde{g}) - \frac{1}{2} \tilde{g}^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - \frac{c^2 M_{\text{Pl}}^2}{32\pi} \left(1 - e^{-\frac{4\sqrt{\pi}}{\sqrt{3}M_{\text{Pl}}}\varphi} \right)^2 \right]. \quad (4.5)$$

The action S_E thus obtained is similar to the corresponding actions in the Einstein frame for the R^2 gravity and Higgs-driven inflation models. Indeed, in all cases the potential has a similar good approximation during inflation, namely

$$V_E \simeq \tilde{C} \left(1 - e^{-\frac{4\sqrt{\pi}}{\sqrt{3}M_{\text{Pl}}}\varphi} \right)^2, \quad (4.6)$$

where the constant \tilde{C} is defined in terms of parameters of the model. As a consequence, we come to the conclusion that for values of parameters C and C_1 connected by (3.6), if we can

find values for the other parameters such that the condition (4.1) is satisfied, and the value of \tilde{C} is close to the value of the corresponding constant in the R^2 gravity and Higgs-driven inflation models, then the inflationary parameters in our model approximately satisfy the relations (1.3). Note that these relations guarantee a good agreement of our inflationary model with Planck data [30]. Also, the same conditions ensure a graceful exit from inflation, similar to the exit that occurs in the above-mentioned, well-known inflationary scenarios.

Now, we consider conditions for the strong coupling regime to be satisfied (4.1). Substituting the function U , given by (3.7), in (4.1), we get

$$\frac{2bn^2\psi^{4n-2}}{\psi^{2n} + 6\xi_0 - 1} \gg 1. \quad (4.7)$$

Using condition (3.10), we obtain

$$\frac{3M_{\text{Pl}}^2\psi^{4n-2}n^2}{4\pi(6\xi_0 - 1)\mu^2(\psi^{2n} + 6\xi_0 - 1)} \gg 1. \quad (4.8)$$

For all values of parameters from Table 2 this inequality is fulfilled and, moreover, the minimal value of the expression on the rhs is higher than 700. For $\mu = 0.1M_{\text{Pl}}$ (see Table 1) the value of this expression lays between 4 and 5. Note that the values of the inflationary parameters for $N_e = 60$ are similar in both tables.

Let us now consider the family of models found in the preceding section, with arbitrary values for the parameters C and C_1 . We do not assume that the condition (3.6) is satisfied. Instead, we assume that the functions U and V are connected by Eq. (4.2), which is one of the conditions for the models to belong to the class of the $\alpha - \beta$ cosmological attractors. We now find the corresponding restrictions on the parameter C_1 . Direct substitution of the functions

$$U = \frac{b}{6}\mu^2 \left(1 + (6\xi_0 - 1)\psi^{Cg^2}\right) \psi^{-2C_1g^2+2} \quad \text{and} \quad V = a_1\kappa_1g^2\mu^4\psi^{4(1-C_1g^2)} \quad (4.9)$$

in the condition (4.2) leads to the equality

$$\left[a_1\kappa_1g^2\mu^4 - \frac{1}{36}\mu^4c^2b^2\right]\psi^{4(1-C_1g^2)} - \frac{(6\xi_0 - 1)}{18}\mu^4c^2b^2\psi^{4+g^2(C-4C_1)} - \frac{(6\xi_0 - 1)^2}{36}\mu^4c^2b^2\psi^{4+2g^2(C-2C_1)} + \frac{1}{3}\mu^2c^2bU_0 \left\{\psi^{2(1-C_1g^2)} + (6\xi_0 - 1)\psi^{2+g^2(C-2C_1)}\right\} = c^2U_0^2.$$

The r.h.s. of this equation is a nonzero constant, hence, the l.h.s. should include ψ in the zeroth degree. This gives a few possible conditions on the parameters C and C_1 . One of these condition is (3.6) that allows to construct inflationary models. Let us check other possibilities, namely, $C_1 = 1/g^2$ and $C_1 = 1/g^2 + C/4$. The former case corresponds to a constant potential V , therefore, condition (4.2) is satisfied for a constant U only. A straightforward calculation shows that, in the latter case, the condition (4.2) is satisfied only for $C = 0$, then this case coincides with the former one. Thus, in both cases V_E is a constant and the inflation scenario is not possible.

Summing up, we arrive to the conclusion that condition (3.6) is both necessary and sufficient in order to get the condition (4.2), which provides a cosmological attractor, known as the $\alpha - \beta$ attractor [49]. And, as a consequence, we obtain an $\alpha - \beta$ attractor only if the

function U includes the Hilbert–Einstein term. But the opposite statement is wrong, because for

$$C_1 = \frac{1}{g^2}, \quad C = \frac{2}{g^2},$$

we get

$$U = \frac{1}{6}b\mu^2 + (6\xi_0 - 1)b\phi^2, \quad V = \tilde{a}\kappa_1 g^2 \mu^4,$$

and the condition (3.6) is not satisfied.

5 Comparison with R^2 gravity

Condition (4.1) means, in fact, that the kinetic term $\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}$ in action (2.1) is negligibly small during inflation. Thus, inflation in the model here considered is very close to the inflation model in $f(R)$ gravity that corresponds to the action (2.1) without kinetic term for the scalar field. This fact is very useful in order to analyse the onset of inflation in the model under consideration.

First of all, for the reasons above, there is actually no difference between the inflationary parameters for the model considered, calculated in the Jordan frame, and the inflationary parameters for the $f(R)$ gravity model, which can be calculated using the Einstein frame. Therefore, if condition (4.1) is satisfied, then the difference between the inflationary parameters calculated in the Jordan and in the Einstein frame, respectively, is negligibly small.

To construct the inflationary model we make use of the linear curvature approximation, neglecting the induced one-loop pure gravitational term, proportional to R^2 . To estimate the importance of this term in the model, we add to action (2.1) the γR^2 term, where γ is a constant. Also, we remove the kinetic term that is negligibly small during inflation and get

$$S_m = \int d^4x \sqrt{-g} [U(\phi)R - V(\phi) + \gamma R^2]. \quad (5.1)$$

Using standard formulae [9], we obtain the corresponding $f(R)$ gravity:

$$S_R = \int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{16\pi} R + \left(\frac{1}{4c^2} + \gamma \right) R^2 \right], \quad (5.2)$$

where the constant c^2 is defined by (4.3). So, the linear curvature approximation is correct under the condition $|\gamma| \ll 1/(4c^2)$. For $\mu = 10^{-3}$ we obtain

$$\frac{1}{4c^2} = \frac{1}{1024\pi^2 \tilde{a} g^2 \kappa_1 (6\xi_0 - 1)^2} \times 10^{12} \simeq \frac{1}{\tilde{a} \kappa_1 g^2 (6\xi_0 - 1)^2} \times 10^8. \quad (5.3)$$

For Starobinsky's R^2 inflation, the coefficient of R^2 is defined by the normalization of the amplitude of the primordial density perturbations, and is of the order of $10^9 M_{\text{Pl}}^2$ (see, for example, [9]). In our model the value of this coefficient gives the condition for their production $\tilde{a}\kappa_1 g^2$.

The coefficient γ that corresponds to the induced one-loop pure gravitational term is much smaller than 10^8 . This is a general result for the inflationary model based on quantum field theory. In Higgs-driven inflation the induced one-loop pure gravitational term, proportional to R^2 , is negligibly small during the inflationary epoch as well [16, 18, 19]. That is why one can neglect the γR^2 term and use the linear curvature approach for the construction of inflationary models.

6 Conclusions

In this paper we have considered the possibility to construct inflationary models starting from a finite gauge model. The tree-level potential that corresponds to the cosmological constant in the Einstein frame is not suitable for inflation. In our previous work [45], we checked for the possibility to construct inflationary scenarios with unstable de Sitter solutions and found that this is not possible for the finite gauge models considered.

Here, we have adopted a different strategy and constructed suitable, and quite natural, inflationary scenarios stemming from very fundamental physical principles, and which are perfectly compatible with the most up to date astronomical data. The widely used, standard procedure to get an inflationary model is to add by hand the Hilbert–Einstein term to the action [17, 18, 46]. We have here shown that generic RG-corrections allow us to generate this term and to get thus, in a natural way, a model of inflation which is compatible with accurate cosmological observations. For the finite gauge model considered, we have shown explicitly that the inflationary scenario is possible, in spite of the absence of an unstable de Sitter solution. We have also found that, for some reasonable values of the parameters, our finite gauge model is in good agreement with the most recent data coming from astronomical observations [29, 30, 35].

The inflationary scenario here devised belongs to the class of $\alpha - \beta$ cosmological attractors, according to the convenient classification given in the literature [49]. The form of the potential in the Einstein frame is quite close to those of the corresponding potentials for Higgs-driven inflation [17] and for Starobinsky’s R^2 inflation [11]. This is a remarkable result, taking into account the fact that our model is derived from fundamental physical principles of quantum field theory, which, moreover, lead to the necessary Hilbert–Einstein term in a generic and natural way.

As is known, the inflationary scenarios mentioned above have parameters which are compatible with the astronomical data and very close to one another. This yields even more value to the fact that, at the same time, there is this noticeable difference that distinguishes our model from both R^2 -inflation and Higgs-driven inflation. The scalar field belongs to the matter sector; moreover, in clear distinction to Higgs-driven inflation, in the model here constructed the Hilbert–Einstein term arises as a result of compulsory quantum corrections at the one-loop approximation and needs not be imposed by hand.

In the paper, we have made explicit comments to the issue of the possible quantum equivalence of the formulations in the Jordan and in the Einstein frames [61, 62]. We have used RG-improved potential which sums all leading logs beyond the one-loop approximation. Of course, RG improved effective action includes also higher-derivative terms, e.g. the R^2 term, which is less relevant for the problem under investigation because it gives the corrections of next-to-leading order. We are going to compare the inflationary models under investigation with the well-known R^2 inflationary models with scalar fields [70] elsewhere.

We have also discussed QFT in curved space-time, in the case where the external gravitational field is a classical one while matter behaves according to QFT. We have considered Quantum Gravity effects to be less relevant in this approach, since we work below the Planck scale but, generally speaking, the same formulation could be applied to perturbatively renormalizable quantum gravity as, for instance, R^2 gravity. The theory under discussion is multiplicatively renormalizable, as explicitly demonstrated in the book [25]. We have used a renormalization group formulation for QFT in curved space-time, following [25], where the one-loop counter terms are explicitly calculated in dimensional regularization. Owing to the

use of dimensional regularization, there is no dependence on the cut-off and, being multiplicatively renormalizable in the external gravitational field, the theory under investigation is a closed one (not an effective theory), since higher-loop corrections repeat the form of the initial action. Specifically, we work with the effective action calculated in the book [25] and then apply the RG improvement procedure in order to get the sum of all leading logs of the perturbation theory. The calculation of the beta-functions is based on the use of dimensional regularization and, hence, it does not depend on any explicit cut-off choice. In short, we have worked with the RG improved effective action of the theory under discussion.

Finally, it would be interesting to further compare the finite gauge inflationary scenario with all these other inflationary models. To do that, we plan to study the transition from inflation to the later stages of the Universe evolution, starting with reheating [71, 72]. As has been shown [53], the reheating temperature can be very different for different models in the same class of cosmological attractors, indeed $T_{reh} = 3.1 \times 10^9 GeV$ for R^2 inflation and $T_{reh} \simeq 6 \times 10^{13} GeV$ (with an uncertainty factor of about two) for Higgs-driven inflation. The study of reheating may also give additional constraints on the parameters of the model here considered. We plan to address the details of the reheating scenario for our class of models in a future publication.

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